Predictive SUSY GUT Model for CP Violation, Fermion Masses and Mixings

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Abstract

CP violation, fermion masses and mixing angles including that of neutrinos are studied in an SUSY SO(10)$\times\Delta(48)\times$ U(1) model. The nonabelian SU(3) discrete family symmetry $\Delta(48)$ associated with a simple scheme of U(1) charge assignment on various fields concerned in superpotential leads to unique Yukawa coupling matrices with zero textures. Thirteen parameters involving masses and mixing angles in the quark and charged lepton sector are successfully predicted by only four parameters. The masses and mixing angles for the neutrino sector could also be predicted by constructing an appropriate heavy Majorana neutrino mass matrix without involving new parameters. It is found that the atmospheric neutrino deficit, the mass limit put by hot dark matter and the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ events may simultaneously be explained, but solar neutrino puzzle can be solved only by introducing a sterile neutrino. An additional parameter is added to obtain the mass and mixing of the sterile neutrino. The hadronic parameters B_K and $f_B\sqrt{B}$ are extracted from the observed K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ mixings respectively. The direct CP violation $(\varepsilon'/\varepsilon)$ in kaon decays and the three angles α, β and γ of the unitarity triangle in the CKM matrix are also presented. More precise measurements of $\alpha_s(M_Z)$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$, m_t, as well as various CP violation and neutrino oscillation experiments will provide an important test for the present model and guide us to a more fundamental theory.

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I. INTRODUCTION

The standard model (SM) is a great success. But 18 phenomenological parameters in the SM have to be introduced to describe all the low energy data in the quark and charged lepton sector, which implies that the SM cannot be the fundamental theory. Furthermore, neutrinos are assumed to be massless in the SM. Thus recent evidence for oscillation of atmospheric neutrinos (and hence nonzero neutrino mass) reported by the Super-Kamiokande collaboration is thought as a major milestone in the search for new physics beyond the standard model(SM). Studies on neutrino physics have resulted in the following observations: i), The Super-Kamiokande data [1] on atmospheric neutrino anomaly provide a strong evidence that neutrinos are massive; ii), The Super-Kamiokande data on solar neutrino [2] cannot yet decisively establish whether the solar neutrino deficit results from MSW solutions [3] with small or large mixing angles or Vacuum oscillation solution though the most recent data favor the MSW solutions with large mixing angle [4]. iii), To describe all the neutrino phenomena such as the atmospheric neutrino anomaly, the solar neutrino deficit and the results from the LSND experiment, it is necessary to introduce a sterile neutrino. It indicates that with only three light neutrinos, one of the experimental data must be modified; iv), The current experimental data cannot establish whether neutrinos are Dirac-type or Majorana-type. The failure of detecting neutrinoless double beta decay only provides, for Majorana-type neutrinos, an upper bound on an 'effective' electron neutrino mass; v), Large neutrino masses around several electron volts may play an important role in the evolution of the large-scale structure of the universe. On the other hand, the 18 parameters in SM have been extracted from various experiments although they are not yet equally well known. Some of them have an accurcy of better than 1%, but some others less than 10%. To improve the accuracy of these parameters and understand them is a big challenge for particle physics. The mass spectrum and the mixing angles observed remind us that we are in a stage similar to that of atomic spectroscopy before Balmer. Much effort has been made along this direction. It was first observed by Gatto *et al*, Cabbibo and Maiani [5] that the Cabbibo angle is close to $\sqrt{m_d/m_s}$. This observation initiated the investigation of the texture structure with zero elements [6] in the fermion Yukawa coupling matrices. The well-known examples are the Fritzsch ansatz [7] and Georgi-Jarlskog texture [8], which has been extensively studied and improved substantially in the literature [9]. Ramond, Robert and Ross [10] presented a general analysis on five symmetric texture structures with zeros in the quark Yukawa coupling matrices. A general analysis and review of the previous studies on the texture structure was given by Raby in [11]. A numerous of papers [12] have investigated some interesting models with texture zeros based on supersymmetric (SUSY) SO(10). A general operator analysis for the quark and charged lepton Yukawa coupling matrices with two zero textures '11' and '13' was made in ref. [13]. Though the texture '22' and '32' are not unique they could fit successfully the 13 observables in the quark and charged lepton sector with only six parameters. We have also shown [14] that the same 13 parameters as well as 10 parameters concerning the neutrino sector (though not unique for this sector) can be successfully described in an SUSY SO(10)$\times\Delta(48)\times$ U(1) model with large $\tan\beta$, where the universality of Yukawa coupling of superpotential was assumed. The resulting texture of mass matrices in the low energy region is quite unique and depends only on a single coupling constant and some vacuum expectation values (VEVs) caused by

necessary symmetry breaking. The 23 parameters were predicted by only five parameters with three of them determined by the symmetry breaking scales of U(1), SO(10), SU(5) and $SU(2)_L$. In that model, the ratio of the VEVs of two light Higgs $\tan\beta \equiv v_2/v_1$ has large value $\tan\beta \sim m_t/m_b$. In general, there exists another interesting solution with small value of $\tan\beta \sim 1$. Such a class of model could also give a consistent prediction on top quark mass and other low energy parameters. Furthermore, models with small value of $\tan\beta \sim 1$ are of phenomenological interest in testing Higgs sector in the minimum supersymmetric standard model (MSSM) at the Colliders [15]. Most of the existing models with small values of $\tan\beta$ in the literature have more parameters than those with large values of $\tan\beta \sim m_t/m_b$. This is because the third family unification condition $\lambda_t^G = \lambda_b^G = \lambda_\tau^G$ has been changed to $\lambda_t^G \neq \lambda_b^G = \lambda_\tau^G$. Besides, some relations between the up-type and down-type quark (or charged lepton) mass matrices have also been lost in the small $\tan\beta$ case when two light Higgs doublets needed for $SU(2)_L$ symmetry breaking belong to different 10s of SO(10). Although models with large $\tan\beta$ have less parameters, large radiative corrections [16] to the bottom quark mass and Cabibbo-Kobayashi-Maskawa (CKM) mixing angles might arise depending on an unkown spectrum of supersymmetric particles.

In a recent Rapid Communication [17], we have presented an alternative model with small value of $\tan\beta \sim 1$ based on the same symmetry group SUSY SO(10)×Δ(48)×U(1) as the model [14] with large value of $\tan\beta$. It is amazing to find out that the model with small $\tan\beta \sim 1$ in [17] has more predictive power on fermion masses and mixings. For convenience, we refer the model in [14] as Model I (with large $\tan\beta \sim m_t/m_b$) and the model in [17,18] as Model II (with small $\tan\beta \sim 1$).

In this paper, we will present in much greater detail an analysis for the model II. Our main considerations can be summarized as follows:

1) SO(10) is chosen as the unification group[1] so that the quarks and leptons in each family are unified into a **16**-dimensional spinor representation of SO(10).

2) The non-abelian dihedral group Δ(48), a subgroup of SU(3) ($\Delta(3n^2)$ with $n = 4$), is taken as the family group[2]. Thus, the three families can be unified into a triplet **16**-dimensional spinor representation of SO(10)×Δ(48). U(1) is family-independent and is introduced to distinguish various fields which belong to the same representations of SO(10)×Δ(48). The irreducible representations of Δ(48) consisting of five triplets and three singlets are found to be sufficient to build interesting texture structures for fermion mass matrices. The symmetry Δ(48)× U(1) naturally ensures the texture structure with zeros for fermion Yukawa coupling matrices. Furthermore, the non-abelian flavor symmetries provides a super-GIM mechanism to supress flavor changing neutral currents induced by supersymmetric particles [22,23,**?**,24].

3) The universality of Yukawa coupling of the superpotential before symmetry breaking is simply assumed to reduce possible free parameters, i.e., all the coupling coefficients in

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[1]Recently, a three-family SO(10) grand unification theory was found in the string theories from orbifold approach [19]. Other possible theories can be found from the free fermionic approach [20].

[2]Recently, the non-abelian discrete symmetry group of a subgroup of U(2) has been taken as the family symmetry and applied to build the unified model [21]

the renormalizable superpotentials are assumed to be equal and have the same origins from perhaps a more fundamental theory. We know in general that universality of charges occurs only in the gauge interactions due to charge conservation like the electric charge of different particles. In the absence of strong interactions, family symmetry could keep the universality of weak interactions in a good approximation after breaking. In the present theory, there are very rich structures above the grand unification theory (GUT) scale with many heavy fermions and scalars and their interactions are taken to be universal before symmetry breaking. All heavy fields must have some reasons to exist and interact which we do not understand at this moment. So that it can only be an ansatz at the present moment since we do not know the answer governing the behavior of nature above the GUT scale. As the Yukawa coupling matrices of the quarks and leptons in the present model are generated at the GUT scale, so that the initial conditions of the renormalization group evaluation for them will be set at the GUT scale[3]. As the resulting Yukawa couplings only rely on the ratios of the coupling constants of the renormalizable superpotentials at the GUT scale, the predictions on the low energy observables will not be affected by the renormalization group (RG) effects running from the Planck scale to the GUT scale as long as the relative value of the ratios for the '22' and '32' textures is unchanged. For this aim, the '22' and '32' textures are constructed in such a way that they have a similar superpotential structure and the fields concerned belong to the same representations of the symmetry group. As we know that the renormalization group evaluation does not change the represenations of a symmetry group, thus the ratios of the coupling constants for the '22' and '32' textures should remain equal at the GUT scale. As we will see below, even if we abondon the general assumption of an unversal coupling for all the Yukawa terms in the superpotential, the above feature can still be ensured by imposing a permutation symmetry among the fields concerning the '22' and '32' textures after family symmetry breaking. As the numerical predictions on the low energy parameters so found are very encouraging and interesting, we believe that there must be a deeper reason that has to be found in the future.

4) The two light Higgs doublets are assumed to belong to an unique 10 representation Higgs of SO(10).

5) Both the symmetry breaking direction of SO(10) down to SU(5) and the two symmetry breaking directions of SU(5) down to $SU(3)_c\times SU(2)_L\times U(1)$ are carefully chosen to ensure the needed Clebsch coefficients for quark and lepton mass matrices . The mass splitting between the up-type quark and down-type quark (or charged lepton) Yukawa couplings is attributed to the Clebsch factors caused by the SO(10) symmetry breaking direction. Thus the third family four-Yukawa coupling relation at the GUT scale will be given by

$$\lambda_b^G = \lambda_\tau^G = \frac{1}{3^n}\lambda_t^G = 5^{n+1}\lambda_{\nu_\tau}^G \tag{1}$$

where the factors $1/3^n$ and 5^{n+1} with n being an integer are the Clebsch factors. A factor

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[3]For models in which the third family Yukawa interaction is considered to be a renormalizble one starting from the Planck scale and the other two family Yukawa interactions are effectively generated at the GUT scale, one then needs to consider the renormalization effect the third family Yukawa coupling from the Planck scale down to the GUT scale.

$1/3^n$ will also multiply the down-type quark and charged lepton Yukawa coupling matrices.

6) CP symmetry is broken spontaneously in the model, a maximal CP violation is assumed to further diminish free parameters.

With the above considerations, the resulting model has found to provide a successful prediction on 13 parameters in the quark and charged lepton sector as well as an interesting prediction on 10 parameters in the neutrino sector with only four parameters. One is the universal coupling constant and the other three are determined by the ratios of vacuum expectation values (VEVs) of the symmetry breaking scales and the RG effects above the GUT scale. One additional parameter resulting from the VEV of a singlet scalar is introduced to obtain the mass and mixing angle of a sterile neutrino. Our paper is organized as follows: In section 2, we will present the results of the Yukawa coupling matrices. The resulting masses and CKM quark mixings are presented in section 3. In section 4 neutrino masses and CKM-type mixings in the lepton sector are presented. All existing neutrino experiments are discussed and shown how they may be understandable in the present model. Conclusions and remarks are presented in the last section.

II. YUKAWA COUPLING MATRICES

With the above considerations, a model based on the symmetry group SUSY SO(10)$\times\Delta(48)\times$ U(1) with a single coupling constant and small value of $\tan\beta$ is constructed. Yukawa coupling matrices which determine the masses and mixings of all quarks and leptons are obtained by carefully choosing the structure of the physical vacuum and integrating out the heavy fermions at the GUT scale. We find

$$\Gamma_u^G = \frac{2}{3}\lambda_H \begin{pmatrix} 0 & \frac{3}{2}z'_u\epsilon_P^2 & 0 \\ \frac{3}{2}z_u\epsilon_P^2 & -3y_u\epsilon_G^2 e^{i\phi} & -\frac{\sqrt{3}}{2}x_u\epsilon_G^2 \\ 0 & -\frac{\sqrt{3}}{2}x_u\epsilon_G^2 & w_u \end{pmatrix} \tag{2}$$

and

$$\Gamma_f^G = \frac{2}{3}\lambda_H \frac{(-1)^{n+1}}{3^n} \begin{pmatrix} 0 & -\frac{3}{2}z'_f\epsilon_P^2 & 0 \\ -\frac{3}{2}z_f\epsilon_P^2 & 3y_f\epsilon_G^2 e^{i\phi} & -\frac{1}{2}x_f\epsilon_G^2 \\ 0 & -\frac{1}{2}x_f\epsilon_G^2 & w_f \end{pmatrix} \tag{3}$$

for $f = d, e$, and

$$\Gamma_\nu^G = \frac{2}{3}\lambda_H \frac{(-1)^{n+1}}{3^n}\frac{1}{5^{n+1}} \begin{pmatrix} 0 & -\frac{15}{2}z'_\nu\epsilon_P^2 & 0 \\ -\frac{15}{2}z_\nu\epsilon_P^2 & 15y_\nu\epsilon_G^2 e^{i\phi} & -\frac{1}{2}x_\nu\epsilon_G^2 \\ 0 & -\frac{1}{2}x_\nu\epsilon_G^2 & w_\nu \end{pmatrix} \tag{4}$$

for Dirac-type neutrino coupling. We will choose $n = 4$ in the following considerations. $\lambda_H = \lambda_H^0 r_3$, $\epsilon_G \equiv (\frac{v_5}{v_{10}})\sqrt{\frac{r_2}{r_3}}$ and $\epsilon_P \equiv (\frac{v_5}{\bar{M}_P})\sqrt{\frac{r_1}{r_3}}$ are three parameters. Where λ_H^0 is a universal coupling constant expected to be of order one, r_1, r_2 and r_3 denote the ratios of the coupling constants of the superpotential at the GUT scale for the textures '12', '22' ('32') and '33' respectively. They represent the possible renormalization group (RG) effects running from the scale $\bar{M}_P$ to the GUT scale. Note that the RG effects for the textures '22'

and '32' are considered to be the same since they are generated from a similar superpotential structure after integrating out the heavy fermions and the fields concerned belong to the same representations of the symmetry group. This can be explicitly seen from their effective operators W_{22} and W_{32} given in eq. (6). $\bar{M}_P$, v_{10} and v_5 are the VEVs for U(1)×Δ(48), SO(10) and SU(5) symmetry breaking respectively. ϕ is the physical CP phase[4] arising from the VEVs. The assumption of maximum CP violation implies that $\phi = \pi/2$. x_f, y_f, z_f, and w_f ($f = u, d, e, \nu$) are the Clebsch factors of SO(10) determined by the directions of symmetry breaking of the adjoints **45**'s. The following three directions have been chosen for symmetry breaking, namely:

$$
\begin{aligned}
&< A_X >= 2v_{10}\ diag.(1,\ 1,\ 1,\ 1,\ 1) \otimes \tau_2, \\
&< A_z >= 2v_5\ diag.(-\frac{1}{3},\ -\frac{1}{3},\ -\frac{1}{3},\ -1,\ -1) \otimes \tau_2, \\
&< A_u >= \frac{1}{\sqrt{3}} v_5\ diag.(2,\ 2,\ 2,\ 1,\ 1) \otimes \tau_2
\end{aligned}
\tag{5}
$$

Their corresponding U(1) hypercharges are given in Table I.

TABLE I. U(1) Hypercharge Quantum Number

	'X'	'u'	'z'	B-L	T_{3R}
q	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
u^c	1	0	$\frac{5}{3}$	-$\frac{1}{3}$	$\frac{1}{2}$
d^c	-3	-$\frac{2}{3}$	-$\frac{7}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$
l	- 3	-1	-1	-1	0
e^c	1	$\frac{2}{3}$	-1	1	-$\frac{1}{2}$
ν^c	5	$\frac{4}{3}$	3	1	$\frac{1}{2}$

The Clebsch factors associated with the symmetry breaking directions can be easily read off from the U(1) hypercharges of the above table. The related effective operators obtained after the heavy fermion pairs integrated out are[5]

$$
\begin{aligned}
W_{33} &= \lambda_H^0 r_3 16_3\ \eta_X \eta_A 10_1 \eta_A \eta_X\ 16_3 \\
W_{32} &= \lambda_H^0 r_2 16_3\ \eta_X \eta_A \left(\frac{A_z}{A_X}\right) 10_1 \left(\frac{A_z}{A_X}\right) \eta_A\ 16_2 \\
W_{22} &= \lambda_H^0 r_2 16_2\ \eta_A \left(\frac{A_u}{A_X}\right) 10_1 \left(\frac{A_u}{A_X}\right) \eta_A\ 16_2 e^{i\phi} \\
W_{12} &= \lambda_H^0 r_1\ 16_1\ [\left(\frac{v_5}{\bar{M}_P}\right)^2 \eta'_A 10_1 \eta'_A \\
&\quad + \left(\frac{v_{10}}{\bar{M}_P}\right)^2 \eta_A \left(\frac{A_u}{A_X}\right) 10_1 \left(\frac{A_z}{A_X}\right) \eta_A]\ 16_2
\end{aligned}
\tag{6}
$$

[4] We have rotated away other possible phases by a phase redefinition of the fermion fields.

[5]Note that W_{22} is slightly modified in comparison with the one in [17] since we have renormalized the VEV $< A_u >$. As a consequence, only the Clebsch factor y_ν is modified, which does not affect all the numerical predictions.

with $n = 4$ and $\phi = \pi/2$. $\eta_A = (v_{10}/A_X)^{n+1}$ and $\eta'_A = (v_{10}/A_X)^{n-3}$. The factor $\eta_X = 1/\sqrt{1+2\eta_A^2}$ in eq. (6) arises from mixing, and provides a factor of $1/\sqrt{3}$ for the up-type quark. It remains almost unity for the down-type quark and charged lepton as well as neutrino due to the suppression of large Clebsch factors in the second term of the square root. The relative phase (or sign) between the two terms in the operator W_{12} has been fixed. The resulting Clebsch factors are

$$
\begin{aligned}
& w_u = w_d = w_e = w_\nu = 1, \\
& x_u = 5/9,\ x_d = 7/27,\ x_e = -1/3,\ x_\nu = 1/5, \\
& y_u = 0,\ y_d = y_e/3 = 2/27,\ y_\nu = 4/225, \\
& z_u = 1,\ z_d = z_e = -27,\ z_\nu = -15^3 = -3375, \\
& z'_u = 1 - 5/9 = 4/9,\ z'_d = z_d + 7/729 \simeq z_d, \\
& z'_e = z_e - 1/81 \simeq z_e,\ z'_\nu = z_\nu + 1/15^3 \simeq z_\nu.
\end{aligned}
\tag{7}
$$

In obtaining the Γ_f^G matrices, some small terms arising from mixings between the chiral fermion 16_i and the heavy fermion pairs $\psi_j(\bar{\psi}_j)$ are neglected. They are expected to change the numerical results no more than a few percent for the up-type quark mass matrix and are negligible for the down-type quark and lepton mass matrices due to the strong suppression of the Clebsch factors. This set of effective operators which lead to the above given Yukawa coupling matrices Γ_f^G is quite unique for a successful prediction on fermion masses and mixings. A general superpotential leading to the above effective operators will be given in section 6. We would like to point out that unlike many other models in which W_{33} is assumed to be a renormalizable interaction before symmetry breaking, the Yukawa couplings of all the quarks and leptons (both heavy and light) in both Model II and Model I are generated at the GUT scale after the breakdown of the family group and SO(10). Therefore, initial conditions for renormalization group (RG) evolution will be set at the GUT scale for all the quark and lepton Yukawa couplings. The hierarchy among the three families is described by the two ratios ϵ_G and ϵ_P. The mass splittings between the quarks and leptons as well as between the up and down quarks are determined by the Clebsch factors of SO(10). From the GUT scale down to low energies, Renormalization Group (RG) evolution has been taken into account. The top-bottom splitting in the present model is mainly attributed to the Clebsch factor $1/3^n$ with $n = 4$ rather than the large value of $\tan\beta$ caused by the hierarchy of the VEVs v_1 and v_2 of the two light Higgs doublets.

An adjoint **45** A_X and a 16-dimensional representation Higgs field Φ ($\bar{\Phi}$) are needed for breaking SO(10) down to SU(5). Adjoint **45** A_z and A_u are needed to break SU(5) further down to the standard model $SU(3)_c\times SU_L(2)\times U(1)_Y$.

III. PREDICTIONS

From the Yukawa coupling matrices given above with $n = 4$ and $\phi = \pi/2$, the 13 parameters in the SM can be determined by only four parameters: a universal coupling constant λ_H and three ratios: ϵ_G, ϵ_P and $\tan\beta = v_2/v_1$. In obtaining physical masses and mixings, renormalization group (RG) effects below the GUT scale has been further taken into consideration. The result at the low energy obtained by scaling down from the GUT

scale will depend on the strong coupling constant α_s. From low-energy measurements [28] and lattice calculations [29], α_s at the scale M_Z, has value around $\alpha_s(M_Z) = 0.113$, which was also found to be consistent with a recent global fit [30] to the LEP data. This value might be reached in nonminimal SUSY GUT models through large threshold effects. As our focus here is on the fermion masses and mixings, we shall not discuss it in this paper. In the present consideration, we take $\alpha_s(M_Z) \simeq 0.113$. The prediction on fermion masses and mixings thus obtained is found to be remarkable. Our numerical predictions are given in Tables II and III with four input parameters, three of them are the well measured charged lepton massess and another is the bottom quark mass.

The predictions on the quark masses and mixings as well as CP-violating effects presented in Table IIb agree remarkably with those extracted from various experimental data. Especially, there are four predictions on $|V_{us}|$, $|V_{ub}/V_{cb}|$, $|V_{td}/V_{ts}|$ and m_d/m_s which are independent of the RG scaling (see eqs. (41)-(44) below).

Let us now analyze in detail the above predictions. To a good approximation, the up-type and down-type quark Yukawa coupling matrices can be diagonalized in the form

$$V_d = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c_d & s_d \\ 0 & -s_d & c_d \end{pmatrix} \tag{8}$$

$$V_u = \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c_u & s_u \\ 0 & -s_u & c_u \end{pmatrix} \tag{9}$$

The CKM matrix at the GUT scale is then given by $V_{CKM} = V_u V_d^\dagger$. Where $s_i \equiv \sin(\theta_i)$ and $c_i \equiv \cos(\theta_i)$ $(i = 1, 2, u, d)$. For $\phi = \pi/2$, the angles θ_i at the GUT scale are given by

$$\tan(\theta_1) \simeq -\frac{z_d}{2y_d}\frac{\epsilon_P^2}{\epsilon_G^2}, \qquad \tan(\theta_2) \simeq \frac{2w_u z_u}{x_u^2}\frac{\epsilon_P^2}{\epsilon_G^4}, \tag{10}$$

$$\tan(\theta_d) \simeq \frac{x_d}{2w_d}\epsilon_G^2, \qquad \tan(\theta_u) \simeq \frac{\sqrt{3}}{2}\frac{x_u}{w_u}\epsilon_G^2. \tag{11}$$

and the Yukawa eigenvalues at the GUT scale are found to be

$$\frac{\lambda_u^G}{\lambda_c^G} = \frac{4w_u^2 z_u z_u'}{x_u^4 \epsilon_G^4}\frac{\epsilon_P^4}{\epsilon_G^4}, \qquad \frac{\lambda_c^G}{\lambda_t^G} = \frac{3}{4}\frac{x_u^2}{w_u^2}\epsilon_G^4, \tag{12}$$

$$\frac{\lambda_d^G}{\lambda_s^G}\left(1 - \frac{\lambda_d^G}{\lambda_s^G}\right)^{-2} = \frac{z_d^2}{4y_d^2}\frac{\epsilon_P^4}{\epsilon_G^4}, \qquad \frac{\lambda_s^G}{\lambda_b^G} = 3\frac{y_d}{w_d}\epsilon_G^2, \tag{13}$$

$$\frac{\lambda_e^G}{\lambda_\mu^G}\left(1 - \frac{\lambda_e^G}{\lambda_\mu^G}\right)^{-2} = \frac{z_e^2}{4y_e^2}\frac{\epsilon_P^4}{\epsilon_G^4}, \qquad \frac{\lambda_\mu^G}{\lambda_\tau^G} = 3\frac{y_e}{w_e}\epsilon_G^2. \tag{14}$$

Using the eigenvalues and angles of these Yukawa matrices , one can easily find the following ten relations among fermion masses and CKM matrix elements at the GUT scale

$$\left(\frac{m_b}{m_\tau}\right)_G = 1, \tag{15}$$

$$\left(\frac{m_s}{m_\mu}\right)_G = \frac{1}{3}, \qquad or \qquad \left(\frac{m_s}{m_b}\right)_G = \frac{1}{3}\left(\frac{m_\mu}{m_\tau}\right)_G \tag{16}$$

$$\left(\frac{m_d}{m_s}\right)_G \left(1-(\frac{m_d}{m_s})_G\right)^{-2} = 9\left(\frac{m_e}{m_\mu}\right)_G \left(1-\left(\frac{m_e}{m_\mu}\right)_G\right)^{-2}, \tag{17}$$

$$\left(\frac{m_t}{m_\tau}\right)_G = 81\ \tan\beta\ , \tag{18}$$

$$\left(\frac{m_c}{m_t}\right)_G = \frac{25}{48}\left(\frac{m_\mu}{m_\tau}\right)_G^2\ , \tag{19}$$

$$\left(\frac{m_u}{m_c}\right)_G = \frac{4}{9}\left(\frac{4}{15}\right)^4 \left(\frac{m_e m_\tau^2}{m_\mu^3}\right)_G\ , \tag{20}$$

$$|\frac{V_{ub}}{V_{cb}}|_G = \tan(\theta_2) = \left(\frac{4}{15}\right)^2 \left(\frac{m_\tau}{m_\mu}\right)_G \sqrt{\left(\frac{m_e}{m_\mu}\right)_G}\ , \tag{21}$$

$$|\frac{V_{td}}{V_{ts}}|_G = \tan(\theta_1) = 3\sqrt{\left(\frac{m_e}{m_\mu}\right)_G}\ , \tag{22}$$

$$|V_{us}|_G = c_1 c_2 \sqrt{\tan^2(\theta_1)+\tan^2(\theta_2)} = 3\sqrt{\left(\frac{m_e}{m_\mu}\right)_G}\left(\frac{1+(\frac{16}{675}(\frac{m_\tau}{m_\mu})_G)^2}{1+9(\frac{m_e}{m_\mu})_G}\right)^{1/2} \tag{23}$$

$$|V_{cb}|_G = c_2 c_d c_u(\tan(\theta_u)-\tan(\theta_d)) = \frac{15\sqrt{3}-7}{15\sqrt{3}}\frac{5}{4\sqrt{3}}\left(\frac{m_\mu}{m_\tau}\right)_G\ . \tag{24}$$

The Clebsch factors in eq. (7) appeared as those miraculus numbers in the above relations. The index 'G' refers throughout to quantities at the GUT scale. The first two relations are well-known in the Georgi-Jarlskog texture. The physical fermion masses and mixing angles are related to the above Yukawa eigenvalues and angles through the renormalization group (RG) equations [31]. As most Yukawa couplings in the present model are much smaller than the top quark Yukawa coupling $\lambda_t^G \sim 1$. In a good approximation, we will only keep top quark Yukawa coupling terms in the RG equations and neglect all other Yukawa coupling terms in the RG equations. The RG evolution will be described by three kinds of scaling factors. Two of them (η_F and R_t) arise from running the Yukawa parameters from the GUT scale down to the SUSY breaking scale M_S which is chosen to be close to the top quark mass, i.e., $M_S \simeq m_t \simeq 170$ GeV, and are defined as

$$m_t(M_S) = \eta_U(M_S)\ \lambda_t^G\ R_t^{-6}\ \frac{v}{\sqrt{2}}\sin\beta\ , \tag{25}$$

$$m_b(M_S) = \eta_D(M_S)\ \lambda_b^G\ R_t^{-1}\ \frac{v}{\sqrt{2}}\cos\beta\ , \tag{26}$$

$$m_i(M_S) = \eta_U(M_S)\ \lambda_i^G\ R_t^{-3}\ \frac{v}{\sqrt{2}}\sin\beta\ , \qquad i=u,c, \tag{27}$$

$$m_i(M_S) = \eta_D(M_S)\lambda_i^G\frac{v}{\sqrt{2}}\cos\beta\ , \qquad i=d,s, \tag{28}$$

$$m_i(M_S) = \eta_E(M_S)\lambda_i^G\frac{v}{\sqrt{2}}\cos\beta\ , \qquad i=e,\mu,\tau, \tag{29}$$

$$\lambda_i(M_S) = \eta_N(M_S)\ \lambda_i^G\ R_t^{-3}\ , \qquad i = \nu_e, \nu_\mu, \nu_\tau\ . \tag{30}$$

with $v = 246$ GeV. $\eta_F(M_S)$ and R_t are given by

$$\eta_F(M_S) = \prod_{i=1}^{3} \left(\frac{\alpha_i(M_G)}{\alpha_i(M_S)} \right)^{c_i^F/2b_i} , \qquad F = U, D, E, N \tag{31}$$

$$R_t^{-1} = e^{-\int_{\ln M_S}^{\ln M_G} (\frac{\lambda_t(t)}{4\pi})^2 dt} = (1 + (\lambda_t^G)^2 K_t)^{-1/12} = \left(1 - \frac{\lambda_t^2(M_S)}{\lambda_f^2} \right)^{1/12} \tag{32}$$

with $c_i^U = (\frac{13}{15}, 3, \frac{16}{3})$, $c_i^D = (\frac{7}{15}, 3, \frac{16}{3})$, $c_i^E = (\frac{27}{15}, 3, 0)$, $c_i^N = (\frac{9}{25}, 3, 0)$, and $b_i = (\frac{33}{5}, 1, -3)$, where λ_f is the fixed point value of λ_t and is given by

$$\lambda_f = \frac{2\pi \eta_U^2}{\sqrt{3I(M_S)}}, \qquad I(M_S) = \int_{\ln M_S}^{\ln M_G} \eta_U^2(t) dt \tag{33}$$

The factor K_t is related to the fixed point value via $K_t = \eta_U^2/\lambda_f^2 = \frac{3I(M_S)}{4\pi^2}$. The numerical value for I taken from Ref. [32] is 113.8 for $M_S \simeq m_t = 170$GeV. λ_f cannot be equal to $\lambda_t(M_S)$ exactly , since that would correspond to infinite λ_t^G , and lead to the so called Landau pole problem at the GUT scale. Other RG scaling factors are derived by running Yukawa couplings below M_S

$$m_i(m_i) = \eta_i\ m_i(M_S), \qquad i = c, b, \tag{34}$$

$$m_i(1GeV) = \eta_i\ m_i(M_S), \qquad i = u, d, s \tag{35}$$

where η_i are the renormalization factors. The physical top quark mass is given by

$$M_t = m_t(m_t) \left(1 + \frac{4}{3} \frac{\alpha_s(m_t)}{\pi} \right) \tag{36}$$

In numerical calculations, we take $\alpha^{-1}(M_Z) = 127.9$, $s^2(M_Z) = 0.2319$, $M_Z = 91.187$ GeV and use the gauge couplings at $M_G \sim 2 \times 10^{16}$ GeV at GUT scale and that of α_1 and α_2 at $M_S \simeq m_t \simeq 170$ GeV

$$\alpha_1^{-1}(m_t) = \alpha_1^{-1}(M_Z) + \frac{53}{30\pi} \ln \frac{M_Z}{m_t} = 58.59, \tag{37}$$

$$\alpha_2^{-1}(m_t) = \alpha_2^{-1}(M_Z) - \frac{11}{6\pi} \ln \frac{M_Z}{m_t} = 30.02, \tag{38}$$

$$\alpha_1^{-1}(M_G) = \alpha_2^{-1}(M_G) = \alpha_3^{-1}(M_G) \simeq 24 \tag{39}$$

we keep $\alpha_3(M_Z)$ as a free parameter in this note. The precise prediction on $\alpha_3(M_Z)$ concerns GUT and SUSY threshold corrections. We shall not discuss it here since our focus in this note is the fermion masses and mixings. Including the three-loop QCD and one-loop QED contributions, the values of η_i in Table IV will be used in numerical calculations.

It is interesting to note that the mass ratios of the charged leptons are almost independent of the RG scaling factors since $\eta_e = \eta_\mu = \eta_\tau$ (up to an accuracy $O(10^{-3})$), namely

$$\frac{m_e}{m_\mu} = \left(\frac{m_e}{m_\mu}\right)_G, \qquad \frac{m_\mu}{m_\tau} = \left(\frac{m_\mu}{m_\tau}\right)_G \tag{40}$$

which is different from the models with large $\tan\beta$. In the present model the τ lepton Yukawa coupling is small. It is easily seen that four relations represented by eqs. (21)-(23) and (17) hold at low energies. Using the known lepton masses $m_e = 0.511$ MeV, $m_\mu = 105.66$ MeV, and $m_\tau = 1.777$ GeV, we obtain four important RG scaling-independent predictions:

$$|V_{us}| = |V_{us}|_G = \lambda \simeq 3\sqrt{\frac{m_e}{m_\mu}}\left(\frac{1+(\frac{16}{675}\frac{m_\tau}{m_\mu})^2}{1+9\frac{m_e}{m_\mu}}\right)^{1/2} = 0.22, \tag{41}$$

$$|\frac{V_{ub}}{V_{cb}}| = |\frac{V_{ub}}{V_{cb}}|_G = \lambda\sqrt{\rho^2+\eta^2} \simeq \left(\frac{4}{15}\right)^2 \frac{m_\tau}{m_\mu}\sqrt{\frac{m_e}{m_\mu}} = 0.083, \tag{42}$$

$$|\frac{V_{td}}{V_{ts}}| = |\frac{V_{td}}{V_{ts}}|_G = \lambda\sqrt{(1-\rho)^2+\eta^2} \simeq 3\sqrt{\frac{m_e}{m_\mu}} = 0.209, \tag{43}$$

$$\frac{m_d}{m_s}\left(1-\frac{m_d}{m_s}\right)^{-2} = 9\frac{m_e}{m_\mu}\left(1-\frac{m_e}{m_\mu}\right)^{-2}, \quad i.e., \quad \frac{m_d}{m_s} = 0.040 \tag{44}$$

and six RG scaling-dependent predictions:

$$|V_{cb}| = |V_{cb}|_G R_t = A\lambda^2 = \frac{15\sqrt{3}-7}{15\sqrt{3}}\frac{5}{4\sqrt{3}}\frac{m_\mu}{m_\tau}R_t = 0.0391\ \left(\frac{0.80}{R_t^{-1}}\right), \tag{45}$$

$$m_s(1GeV) = \frac{1}{3}m_\mu\frac{\eta_s}{\eta_\mu}\eta_{D/E} = 159.53\ \left(\frac{\eta_s}{2.2}\right)\left(\frac{\eta_{D/E}}{2.1}\right)\ MeV, \tag{46}$$

$$m_b(m_b) = m_\tau\frac{\eta_b}{\eta_\tau}\eta_{D/E}R_t^{-1} = 4.25\ \left(\frac{\eta_b}{1.49}\right)\left(\frac{\eta_{D/E}}{2.04}\right)\left(\frac{R_t^{-1}}{0.80}\right)\ GeV\ , \tag{47}$$

$$m_u(1GeV) = \frac{5}{3}(\frac{4}{45})^3\frac{m_e}{m_\mu}\eta_u R_t^3 m_t = 4.23\ \left(\frac{\eta_u}{2.2}\right)\left(\frac{0.80}{R_t^{-1}}\right)^3\left(\frac{m_t(m_t)}{174GeV}\right)\ MeV\ , \tag{48}$$

$$m_c(m_c) = \frac{25}{48}(\frac{m_\mu}{m_\tau})^2\eta_c R_t^3 m_t = 1.25\ \left(\frac{\eta_c}{2.0}\right)\left(\frac{0.80}{R_t^{-1}}\right)^3\left(\frac{m_t(m_t)}{174GeV}\right)\ GeV, \tag{49}$$

$$\begin{aligned} m_t(m_t) &= \frac{\eta_U}{\sqrt{K_t}}\sqrt{1-R_t^{-12}}\frac{v}{\sqrt{2}}\sin\beta \\ &= 174.9\ \left(\frac{\sin\beta}{0.92}\right)\left(\frac{\eta_U}{3.33}\right)\left(\sqrt{\frac{8.65}{K_t}}\right)\left(\frac{\sqrt{1-R_t^{-12}}}{0.965}\right)\ GeV \end{aligned} \tag{50}$$

We have used the fixed point property for the top quark mass. These predictions depend on two parameters R_t and $\sin\beta$ (or λ_t^G and $\tan\beta$). In general, the present model contains four parameters: ϵ_G, ϵ_P, $\tan\beta = v_2/v_1$, and $\lambda_t^G = 81\lambda_b^G = 81\lambda_\tau^G = \frac{2}{3}\lambda_H$. It is not difficult to notice that ϵ_G and ϵ_P are determined solely by the Clebsch factors and mass ratios of the charged leptons

$$\epsilon_G = \left(\frac{v_5}{v_{10}}\right)\sqrt{\left(\frac{r_2}{r_3}\right)} = \sqrt{\frac{m_\mu}{m_\tau}\frac{\eta_\tau}{\eta_\mu}\frac{w_e}{3y_e}} = 2.987\times 10^{-1}, \tag{51}$$

$$\epsilon_P = \left(\frac{v_5}{\bar{M}_P}\right)\sqrt{\left(\frac{r_1}{r_3}\right)} = \left(\frac{4}{9}\frac{m_e m_\mu}{m_\tau^2}\frac{\eta_\tau^2}{\eta_e\eta_\mu}\frac{w_e^2}{z_e^2}\right)^{1/4} = 1.011\times 10^{-2}. \tag{52}$$

The coupling λ_t^G (or R_t) can be determined by the mass ratio of the bottom quark and τ lepton

$$\lambda_t^G = \frac{1}{\sqrt{K_t}} \frac{\sqrt{1-R_t^{-12}}}{R_t^{-6}} = 1.25\ \zeta_t, \tag{53}$$

$$\zeta_t \equiv \left(\sqrt{\frac{8.65}{K_t}}\right) \left(\frac{0.80}{R_t^{-1}}\right)^6 \left(\frac{\sqrt{1-R_t^{-12}}}{0.965}\right), \tag{54}$$

$$R_t^{-1} = \frac{m_b}{m_\tau} \frac{\eta_\tau}{\eta_b} \frac{1}{\eta_{D/E}} = 0.80\ \left(\frac{m_b(m_b)}{4.25GeV}\right) \left(\frac{1.49}{\eta_b}\right) \left(\frac{2.04}{\eta_{D/E}}\right). \tag{55}$$

$\tan\beta$ is fixed by the τ lepton mass

$$\cos\beta = \frac{m_\tau\sqrt{2}}{\eta_E \eta_\tau v \lambda_\tau^G} = \left(\frac{0.41}{\zeta_t}\right) \left(\frac{3^n}{81}\right),$$

$$\sin\beta = \sqrt{1-(\frac{0.41}{\zeta_t}\frac{3^n}{81})^2} = 0.912\ \left(\frac{\sqrt{1-(\frac{0.41}{\zeta_t}\frac{3^n}{81})^2}}{0.912}\right),$$

$$\tan\beta = 2.225\ \left(\frac{81}{3^n}\right) \left(\frac{\sqrt{\zeta_t^2-(0.41)^2(3^n/81)^2}}{0.912}\right). \tag{56}$$

With these considerations, the top quark mass is given by

$$m_t(m_t) = 173.4\ \left(\frac{\eta_U}{3.33}\right) \left(\sqrt{\frac{8.65}{K_t}}\right) \left(\frac{\sqrt{1-R_t^{-12}}}{0.965}\right) \left(\frac{\sqrt{1-(0.41/\zeta_t)^2}}{0.912}\right)\ GeV \tag{57}$$

Given ϵ_G and ϵ_P as well λ_t^G, the Yukawa coupling matrices of the fermions at the GUT scale are then known. It is of interest to expand the above fermion Yukawa coupling matrices Γ_f^G in terms of the parameter $\lambda = 0.22$ (the Cabbibo angle), as Wolfenstein [33] did for the CKM mixing matrix.

$$\Gamma_u^G = 1.25\zeta_t \begin{pmatrix} 0 & 0.60\lambda^6 & 0 \\ 1.35\lambda^6 & 0 & -0.89\lambda^2 \\ 0 & -0.89\lambda^2 & 1 \end{pmatrix}, \tag{58}$$

$$\Gamma_d^G = -\frac{1.25\zeta_t}{81} \begin{pmatrix} 0 & 1.77\lambda^4 & 0 \\ 1.77\lambda^4 & 0.41\lambda^2 e^{i\frac{\pi}{2}} & -1.09\lambda^3 \\ 0 & -1.09\lambda^3 & 1 \end{pmatrix}, \tag{59}$$

$$\Gamma_e^G = -\frac{1.25\zeta_t}{81} \begin{pmatrix} 0 & 1.77\lambda^4 & 0 \\ 1.77\lambda^4 & 1.23\lambda^2 e^{i\frac{\pi}{2}} & 1.40\lambda^2 \\ 0 & 1.40\lambda^2 & 1 \end{pmatrix}, \tag{60}$$

$$\Gamma_\nu^G = -\left(\frac{1.25\zeta_t}{81}\right) \left(\frac{2.581}{5^5}\right) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0.86\lambda^3 e^{i\frac{\pi}{2}} & 1.472\lambda^4 \\ 0 & 1.472\lambda^4 & 1.757\lambda \end{pmatrix} \tag{61}$$

Using the CKM parameters and quark masses predicted in the present model, the bag parameter B_K can be extracted from the indirect CP-violating parameter $|\varepsilon_K| = 2.6 \times 10^{-3}$ in K^0-$\bar{K}^0$ system via

$$B_K = 0.90 \ \left(\frac{0.57}{\eta_2}\right)\left(\frac{|\varepsilon_K|}{2.6\times 10^{-3}}\right)\left(\frac{0.138 y_t^{1.55}}{A^4(1-\rho)\eta}\right)\left(\frac{1.41}{1+\frac{0.246 y_t^{1.34}}{A^2(1-\rho)}}\right) \tag{62}$$

The B-meson decay constant can also be obtained from fitting the B^0-$\bar{B}^0$ mixing

$$f_B\sqrt{B} = 207 \ \left(\sqrt{\frac{0.55}{\eta_B}}\right)\left(\frac{\Delta M_{B_d}(ps^{-1})}{0.465}\right)\left(\frac{0.77 y_t^{0.76}}{A\sqrt{(1-\rho)^2+\eta^2}}\right) \ MeV \tag{63}$$

with $y_t = 175GeV/m_t(m_t)$ and η_2 and η_B being the QCD corrections [34]. Note that we did not consider the possible contributions to ε_K and ΔM_{B_d} from box diagrams through exchanges of superparticles. To have a complete analysis, these contributions should be included in a more detailed consideration in the future. The parameter B_K was estimated ranging from 1/3 to 1 based on various approaches. Recent analysis using the lattice methods [35,36] gives $B_K = 0.82 \pm 0.1$. There are also various calculations on the parameter f_{B_d}. From the recent lattice analyses [35,37], $f_{B_d} = (200 \pm 40)$ MeV, $B_{B_d} = 1.0 \pm 0.2$. QCD sum rule calculations [38] also gave a compatible result. An interesting upper bound [39] $f_B\sqrt{B} <$ 213MeV for $m_c = 1.4$GeV and $m_b = 4.6$ GeV or $f_B\sqrt{B} < 263$MeV for $m_c = 1.5$GeV and $m_b = 5.0$ GeV has been obtained by relating the hadronic mixing matrix element, Γ_{12}, to the decay rate of the bottom quark.

The direct CP-violating parameter Re$(\varepsilon'/\varepsilon)$ in the K-system has been estimated by the standard method. The uncertanties mainly arise from the hadronic matrix elements [40]. We have included the next-to-leading order contributions from the chiral-loop [41–43] and the next-to-leading order perturbative contributions [44,45] to the Wilson coefficients together with a consistent analysis of the $\Delta I = 1/2$ rule. Experimental results on Re$(\varepsilon'/\varepsilon)$ is inconclusive. The NA31 collaboration at CERN reported a value Re$(\varepsilon'/\varepsilon) = (2.3 \pm 0.7)\cdot 10^{-3}$ [46] which clearly indicates direct CP violation, while the value given by E731 at Fermilab, Re$(\varepsilon'/\varepsilon) = (0.74 \pm 0.59)\cdot 10^{-3}$ [47] is compatible with superweak theories [51] in which $\varepsilon'/\varepsilon = 0$. The average value quoted in [27] is Re$(\varepsilon'/\varepsilon) = (1.5 \pm 0.8)\cdot 10^{-3}$[6]

For predicting physical observables, it is better to use J_{CP} , the rephase-invariant CP-violating quantity, together with α, β and γ , the three angles of the unitarity triangle of a three-family CKM matrix

$$\alpha = arg.\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta = arg.\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma = arg.\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \tag{64}$$

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[6] Recently, two improved experiments have reported new results: $Re(\varepsilon'/\varepsilon) = (18.5 \pm 4.5 \pm 5.8) \times 10^{-4}$ by the NA48 collaboration at CERN [48], and $Re(\varepsilon'/\varepsilon) = (28.0 \pm 3.0 \pm 2.8) \times 10^{-4}$ by the KTeV collaboration at Fermilab [49], where only 23% data have been analyzed. The hadronic matrix elements have been reanalyzed with paying attention to the matching between QCD and Chiral Perturbation Theory (ChPT), as a consequence, both direct CP-violating parameter ε'/ε and $\Delta I = 1/2$ rule can be consistently predicted [50]. With the improved hadronic matrix elements, the present predicted values for the mixing angles and CP-violating phase lead to a bigger value for the ratio ε'/ε, numerically, it is found to be $\varepsilon'/\varepsilon \simeq 28 \times 10^{-4}$.

where $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$ can in principle be measured in $B^0/\bar{B}^0 \to \pi^+\pi^-$ [52], $J/\psi K_S$ [53] and $B^- \to K^- D$ [54], respectively. $|V_{us}|$ has been extracted with good accuracy from $K \to \pi e \nu$ and hyperon decays [27]. $|V_{cb}|$ can be determined from both exclusive and inclusive semileptonic B decays with values given by

$$|V_{cb}| = \begin{cases} 0.0406 \pm 0.00036\ (exp.) \pm 0.0016\ (theor.); & \text{ICHEP2000 [55]}\ , \\ 0.0389 \pm 0.0005_{exp} \pm 0.0020_{theor.}; & \text{HQEFT [56,57]} \end{cases} \tag{65}$$

from inclusive semileptonic B decays and

$$|V_{cb}| = \begin{cases} 0.0418 \pm 0.0016\ (exp.) \pm 0.0021\ (theor.); & [55] \\ 0.0404 \pm 0.0015\ (exp.) \pm 0.0020\ (theor.); & \\ 0.0395 \pm 0.0029\ (exp.) \pm 0.0019\ (theor.); & \text{HQEFT [58]} \\ 0.0382 \pm 0.0041\ (exp.) \pm 0.0028\ (theor.); & \text{HQEFT [58]} \end{cases} \tag{66}$$

from exclusive semileptonic B decays. Where the first result was be obtained for $F(1) = 0.88 \pm 0.04$ and the second for $F(1) = 0.91 \pm 0.04$. The third one is from $B \to D^* l\nu$ and the last one from $B \to Dl\nu$.

Another CKM parameter $|V_{ub}/V_{cb}|$ is extracted from a study of the semileptonic B decays near the end point region of the lepton spectrum. The present experimental measurements are compatible with [55,56]

$$|\frac{V_{ub}}{V_{cb}}| = 0.08 \pm 0.01\ (exp.) \pm 0.02\ (theor.) \tag{67}$$

The CKM parameter $|V_{td}/V_{ts}|$ is constrained by the indirect CP-violating parameter $|\varepsilon|$ in kaon decays and B^0-$\bar{B}^0$ mixing x_d. Large uncertainties of $|V_{td}/V_{ts}|$ are caused by the bag parameter B_K and the leptonic B decay constant f_B.

TABLE II. Output observables and model parameters and their predicted values with input parameters $m_e = 0.511$ MeV, $m_\mu = 105.66$ MeV, $m_\tau = 1.777$ GeV and $m_b(m_b) = 4.25$ GeV.

Output parameters	Output values	Data [25–27]	Output para.	Output values
M_t [GeV]	182	175 ± 6	$J_{CP} = A^2\lambda^6\eta$	2.68×10^{-5}
$m_c(m_c)$ [GeV]	1.27	1.27 ± 0.05	α	$86.28°$
m_u(1GeV) [MeV]	4.31	4.75 ± 1.65	β	$22.11°$
m_s(1GeV) [MeV]	156.5	165 ± 65	γ	$71.61°$
m_d(1GeV) [MeV]	6.26	8.5 ± 3.0	m_{ν_τ} [eV]	2.4504
$\lvert V_{us}\rvert = \lambda$	0.22	0.221 ± 0.003	m_{ν_μ} [eV]	2.4498
$\frac{\lvert V_{ub}\rvert}{\lvert V_{cb}\rvert} = \lambda\sqrt{\rho^2+\eta^2}$	0.083	0.08 ± 0.03	m_{ν_e} [eV]	1.27×10^{-3}
$\frac{\lvert V_{td}\rvert}{\lvert V_{ts}\rvert} = \lambda\sqrt{(1-\rho)^2+\eta^2}$	0.209	0.24 ± 0.11	m_{ν_s} [eV]	2.8×10^{-3}
$\lvert V_{cb}\rvert = A\lambda^2$	0.0393	0.039 ± 0.005	$\lvert V_{\nu_\mu e}\rvert$	-0.049
λ_t^G	1.30	-	$\lvert V_{\nu_e \tau}\rvert$	0.000
$\tan\beta = v_2/v_1$	2.33	-	$\lvert V_{\nu_\tau e}\rvert$	-0.049
ϵ_G	0.2987	-	$\lvert V_{\nu_\mu \tau}\rvert$	-0.707
ϵ_P	0.0101	-	$\lvert V_{\nu_e s}\rvert$	0.038
B_K	0.90	0.82 ± 0.10	M_{N_1} [GeV]	~ 333
$f_B\sqrt{B}$ [MeV]	207	200 ± 70	M_{N_2} [GeV]	1.63×10^6
$\mathrm{Re}(\varepsilon'/\varepsilon)/10^{-3}$	1.4 ± 1.0	1.5 ± 0.8	M_{N_3} [GeV]	333

TABLE III. Output observables and model parameters and their predicted values with input parameters $m_e = 0.511$ MeV, $m_\mu = 105.66$ MeV, $m_\tau = 1.777$ GeV and $m_b(m_b) = 4.32$ GeV.

Output parameters	Output values	Data [25–27]	Output para.	Output values
M_t [GeV]	179	175 ± 6	$J_{CP} = A^2\lambda^6\eta$	2.62×10^{-5}
$m_c(m_c)$ [GeV]	1.21	1.27 ± 0.05	α	86.28°
m_u(1GeV) [MeV]	4.11	4.75 ± 1.65	β	22.11°
m_s(1GeV) [MeV]	156.5	165 ± 65	γ	71.61°
m_d(1GeV) [MeV]	6.26	8.5 ± 3.0	m_{ν_τ} [eV]	2.4504
$\|V_{us}\| = \lambda$	0.22	0.221 ± 0.003	m_{ν_μ} [eV]	2.4498
$\frac{\|V_{ub}\|}{\|V_{cb}\|} = \lambda\sqrt{\rho^2+\eta^2}$	0.083	0.08 ± 0.03	m_{ν_e} [eV]	1.27×10^{-3}
$\frac{\|V_{td}\|}{\|V_{ts}\|} = \lambda\sqrt{(1-\rho)^2+\eta^2}$	0.209	0.24 ± 0.11	m_{ν_s} [eV]	2.8×10^{-3}
$\|V_{cb}\| = A\lambda^2$	0.0389	0.039 ± 0.005	$\|V_{\nu_\mu e}\|$	-0.049
λ_t^G	1.20	-	$\|V_{\nu_e\tau}\|$	0.000
$\tan\beta = v_2/v_1$	2.12	-	$\|V_{\nu_\tau e}\|$	-0.049
ϵ_G	0.2987	-	$\|V_{\nu_\mu\tau}\|$	-0.707
ϵ_P	0.0101	-	$\|V_{\nu_e s}\|$	0.038
B_K	0.96	0.82 ± 0.10	M_{N_1} [GeV]	~ 361
$f_B\sqrt{B}$ [MeV]	212	200 ± 70	M_{N_2} [GeV]	1.77×10^6
$\mathrm{Re}(\varepsilon'/\varepsilon)/10^{-3}$	1.4 ± 1.0	1.5 ± 0.8	M_{N_3} [GeV]	361

TABLE IV. Values of η_i and η_F as a function of the strong coupling $\alpha_s(M_Z)$

$\alpha_s(M_Z)$	0.110	0.113	0.115	0.117	0.120
$\eta_{u,d,s}$	2.08	2.20	2.26	2.36	2.50
η_c	1.90	2.00	2.05	2.12	2.25
η_b	1.46	1.49	1.50	1.52	1.55
$\eta_{e,\mu,\tau}$	1.02	1.02	1.02	1.02	1.02
η_U	3.26	3.33	3.38	3.44	3.50
$\eta_D/\eta_E \equiv \eta_{D/E}$	2.01	2.06	2.09	2.12	2.16
η_E	1.58	1.58	1.58	1.58	1.58
η_N	1.41	1.41	1.41	1.41	1.41

A detail analysis of neutrino masses and mixings will be presented in the next section. Before proceeding further, we would like to address the following points: Firstly, given $\alpha_s(M_Z)$ and $m_b(m_b)$, the value of $\tan\beta$ depends, as one sees from eq.(56), on the choice of the integer 'n' in an over all factor $1/3^n$, so do the masses of all the up-type quarks (see eqs. (48)-(50)). For $n > 4$, the value of $\tan\beta$ becomes too small, as a consequence, the resulting top quark mass will be below the present experimental lower bound, so do the masses of the up and charm quarks. In contrast, for $1 < n < 4$, the values of $\tan\beta$ will become larger, the resulting charm quark mass will be above the present upper bound and the top quark mass is very close to the present upper bound. Secondly, given $m_b(m_b)$ and integer 'n', all other quark masses increase with $\alpha_s(M_Z)$. This is because the RG scaling factors η_i and R_t increase with $\alpha_s(M_Z)$. When $\alpha_s(M_Z)$ is larger than 0.117 and n=4, either charm quark mass or bottom quark mass will be above the present upper bound. Finally, the symmetry breaking direction of the adjoint **45** A_z or the Clebsch factor x_u is strongly restricted by both

$|V_{ub}|/|V_{cb}|$ and charm quark mass $m_c(m_c)$. From these considerations, we conclude that the best choice of n will be 4 for small $\tan\beta$ and the value of α_s should around $\alpha_s(M_Z) \simeq 0.113$, which can be seen from table 2b.

IV. NEUTRINO MASSES AND MIXINGS

Neutrino masses and mixings , if they exist, are very important in astrophysics and crucial for model building. Many unification theories predict a see-saw type mass [59] $m_{\nu_i} \sim m_{u_i}^2/M_N$ with $u_i = u, c, t$ being up-type quarks. For $M_N \simeq (10^{-3} \sim 10^{-4})M_{GUT} \simeq 10^{12} - 10^{13}$ GeV, one has

$$m_{\nu_e} < 10^{-7}eV, \qquad m_{\nu_\mu} \sim 10^{-3}eV, \qquad m_{\nu_\tau} \sim (3-21)eV \tag{68}$$

In this case solar neutrino anomalous could be explained by $\nu_e \to \nu_\mu$ oscillation, and the mass of ν_τ is in the range relevant to hot dark matter. However, LSND events and atmospheric neutrino deficit can not be explained in this scenario.

By choosing Majorana type Yukawa coupling matrix differently, one can construct many models of neutrino mass matrix. As we have shown in the Model I that by choosing an appropriate texture structure with some diagonal zero elements in the right-handed Majorana mass matrix, one can explain the recent LSND events, atmospheric neutrino deficit and hot dark matter, however, the solar neutrino anomalous can only be explained by introducing a sterile neutrino. A similar consideration can be applied to the present model. The following texture structure with zeros is found to be interesting for the present model

$$M_N^G = M_R \begin{pmatrix} 0 & 0 & \frac{1}{2}z_N\epsilon_P^2 e^{i(\delta_\nu+\phi_3)} \\ 0 & y_N e^{2i\phi_2} & 0 \\ \frac{1}{2}z_N\epsilon_P^2 e^{i(\delta_\nu+\phi_3)} & 0 & w_N\epsilon_P^4 e^{2i\phi_3} \end{pmatrix} \tag{69}$$

The corresponding effective operators are given by

$$\begin{aligned} W_{13}^N &= \lambda_1^N 16_1 (\frac{A_z}{v_5})(\frac{\bar{\Phi}}{v_{10}})(\frac{\bar{\Phi}}{v_{10}})(\frac{A_u}{v_5})16_3\ e^{i(\delta_\nu+\phi_3)} \\ W_{22}^N &= \lambda_2^N 16_2 (\frac{A_z}{A_X})(\frac{\bar{\Phi}}{v_{10}})(\frac{\bar{\Phi}}{v_{10}})(\frac{A_z}{A_X})16_2\ e^{2i\phi_2} \\ W_{33}^N &= \lambda_3^N 16_3 (\frac{A_u}{v_5})(\frac{\bar{\Phi}}{v_{10}})(\frac{\bar{\Phi}}{v_{10}})(\frac{A_u}{v_5})16_3\ e^{2i\phi_3} \end{aligned}$$

with $M_R = \lambda_H \epsilon_P^4 \epsilon_G^2 v_{10}^2/\bar{M}_P$, $\lambda_1^N = \epsilon_P^2 M_R$, $\lambda_2^N = M_R/\epsilon_G^2$ and $\lambda_3^N = \epsilon_P^4 M_R$. It is not difficult to read off the Clebsch factors

$$y_N = 9/25, \qquad z_N = 4, \qquad w_N = 16/9 \tag{70}$$

where δ_ν, ϕ_2 and ϕ_3 are three phases. For convenience , we first redefine the phases of the three right-handed neutrinos $\nu_{R1} \to e^{i\delta_\nu}\nu_{R1}$, $\nu_{R2} \to e^{i\phi_2}\nu_{R2}$, and $\nu_{R3} \to e^{i\phi_3}\nu_{R3}$, so that the matrix M_N^G becomes real.

The light neutrino mass matrix is then given via see-saw mechanism as follows

$$M_\nu = \Gamma_\nu^G (M_N^G)^{-1} (\Gamma_\nu^G)^\dagger v_2^2/2R_t^{-6}\eta_N^2$$
$$= M_0 \begin{pmatrix} 1.027\lambda^5 & -0.88\lambda^8 e^{i\frac{\pi}{2}} & 1.51\lambda^9 \\ -0.88\lambda^8 e^{-i\frac{\pi}{2}} & 0.37\lambda^4 \cos\delta_\nu - 0.456\lambda^5 & e^{i\delta_\nu} \\ 1.51\lambda^9 & e^{-i\delta_\nu} & 0.49\lambda^{12} \end{pmatrix} \tag{71}$$

with

$$M_0 = \left(\frac{2}{15^5}\right)^2 \left(\frac{15}{\epsilon_P^5}\right) \left(\frac{-w_\nu z_\nu}{y_N z_N}\right) \left(\frac{v_2^2}{2v_5}\right) / R_t^{-6}\eta_N^2 \lambda_H$$
$$= 2.45 \ \left(\frac{2.36 \times 10^{16} GeV}{v_5}\right) \left(\frac{\zeta_t}{1.04}\right) \ eV \tag{72}$$

It is seen that only one phase, δ_ν , is physical. We shall assume again maximum CP violation with $\delta_\nu = \pi/2$. Neglecting the small terms of order above $O(\lambda^7)$, the neutrino mass matrix can be simply diagonalized by

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\nu & -s_\nu \\ 0 & s_\nu & c_\nu \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \tag{73}$$

and the charged lepton mass matrix by

$$V_e = \begin{pmatrix} \bar{c}_1 & -\bar{s}_1 & 0 \\ \bar{s}_1 & \bar{c}_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i & 0 & 0 \\ 0 & c_e & -s_e \\ 0 & s_e & c_e \end{pmatrix} \tag{74}$$

The CKM-type lepton mixing matrix is then given by

$$V_{LEP} = V_\nu V_e^\dagger = \begin{pmatrix} V_{\nu_e e} & V_{\nu_e \mu} & V_{\nu_e \tau} \\ V_{\nu_\mu e} & V_{\nu_\mu \mu} & V_{\nu_\mu \tau} \\ V_{\nu_\tau e} & V_{\nu_\tau \mu} & V_{\nu_\tau \tau} \end{pmatrix} \tag{75}$$
$$= \begin{pmatrix} \bar{c}_1 & \bar{s}_1 & 0 \\ -\bar{s}_1(c_\nu c_e + s_\nu s_e e^{i\delta_\nu}) & \bar{c}_1(c_\nu c_e + s_\nu s_e e^{i\delta_\nu}) & -(s_\nu c_e - c_\nu s_e e^{i\delta_\nu}) \\ -\bar{s}_1(s_\nu c_e - c_\nu s_e e^{i\delta_\nu}) & \bar{c}_1(s_\nu c_e - c_\nu s_e e^{i\delta_\nu}) & c_\nu c_e + s_\nu s_e e^{i\delta_\nu} \end{pmatrix}$$

where the angles are found to be

$$\tan\bar{\theta}_1 = \sqrt{\frac{m_e}{m_\mu}} = 0.0695 \tag{76}$$
$$\tan\theta_e = -\frac{x_e}{2w_e}\epsilon_G^2 = -\frac{m_\mu}{m_\tau}\frac{x_e}{6y_e} = 0.0149 \tag{77}$$
$$\tan\theta_\nu = 1 \tag{78}$$

It is of interest to note that these predictions are solely determined without involving any new parameters.

For masses of light Majorana neutrinos we have

$$m_{\nu_e} = -\frac{1}{4}\frac{z_\nu}{w_\nu} z_N M_0 = 1.27 \times 10^{-3}\ eV, \tag{79}$$

$$m_{\nu_\mu} = \left(1 + \frac{15}{2}\frac{z_\nu w_N}{w_\nu z_N}\epsilon_P^4\right) M_0 \simeq 2.4498\ eV \tag{80}$$

$$m_{\nu_\tau} = \left(1 - \frac{15}{2}\frac{z_\nu w_N}{w_\nu z_N}\epsilon_P^4\right) M_0 \simeq 2.4504\ eV \tag{81}$$

The three heavy Majorana neutrinos have masses

$$M_{N_1} \simeq M_{N_3} \simeq \frac{1}{2} y_N z_N \epsilon_P^7 v_5 \lambda_H \simeq 333\ \left(\frac{v_5}{2.36 \times 10^{16} GeV}\right)\ GeV \tag{82}$$

$$M_{N_2} = y_N \epsilon_P^5 v_5 \lambda_H = 1.63 \times 10^6\ \left(\frac{v_5}{2.36 \times 10^{16} GeV}\right)\ GeV \tag{83}$$

The RG effects above the GUT scale may be absorbed into the mass M_0. The three heavy Majorana neutrinos in the present model have their masses much below the GUT scale, unlike many other GUT models with corresponding masses near the GUT scale. In fact, two of them have masses in the range comparable with the electroweak scale.

As the masses of the three light neutrinos are very small, a direct measurement for their masses would be too difficult. An efficient detection on light neutrino masses can be achieved through their oscillations. The probability that an initial ν_α of energy E (in unit MeV) gets converted to a ν_β after travelling a distance L (in unit m) is

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} V_{\alpha i} V^*_{\beta i} V_{\beta j} V^*_{\alpha j} \sin^2(\frac{1.27 L \Delta m^2_{ij}}{E}) \tag{84}$$

with $\Delta m^2_{ij} = m^2_j - m^2_i$ (in unit eV^2). From the above results, we observe the following

1. a $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_\tau(\bar{\nu}_\tau)$ long-wave length oscillation with

$$\Delta m^2_{\mu\tau} = m^2_{\nu_\tau} - m^2_{\nu_\mu} \simeq 2.9 \times 10^{-3} eV^2\ , \qquad \sin^2 2\theta_{\mu\tau} \simeq 0.987\ , \tag{85}$$

which could explain the atmospheric neutrino with the best fit [1]

$$\Delta m^2_{\mu\tau} = m^2_{\nu_\tau} - m^2_{\nu_\mu} \simeq 3.0 \times 10^{-3} eV^2\ , \qquad \sin^2 2\theta_{\mu\tau} \simeq 1.0\ ; \tag{86}$$

2. a $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$ short wave-length oscillation with

$$\Delta m^2_{e\mu} = m^2_{\nu_\mu} - m^2_{\nu_e} \simeq 6\ eV^2, \qquad \sin^2 2\theta_{e\mu} \simeq 1.0 \times 10^{-2}\ , \tag{87}$$

could explain the LSND experiment [61]

3. Two massive neutrinos ν_μ and ν_τ with

$$m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2.45\ eV\ , \tag{88}$$

fall in the range required by possible hot dark matter [63].

4. $(\nu_\mu - \nu_\tau)$ oscillation will be beyond the reach of CHORUS/NOMAD and E803. However, $(\nu_e - \nu_\tau)$ oscillation may become interesting as a short wave-length oscillation with

$$\Delta m^2_{e\tau} = m^2_{\nu_\tau} - m^2_{\nu_e} \simeq 6\ eV^2, \qquad \sin^2 2\theta_{e\tau} \simeq 1.0 \times 10^{-2}\ , \tag{89}$$

which should provide an independent test on the pattern of the present Majorana neutrino mass matrix.

5. Majorana neutrino allows neutrinoless double beta decay ($\beta\beta_{0\nu}$) [64]. Its decay amplitude is known to depend on the masses of Majorana neutrinos m_{ν_i} and the lepton mixing matrix elements V_{ei}. The present model is compatible with the present experimental upper bound on neutrinoless double beta decay

$$\bar{m}_{\nu_e} = \sum_{i=1}^{3}[V_{ei}^2 m_{\nu_i}\zeta_i] \simeq 1.18\times 10^{-2}\ eV \ < \ \bar{m}_{\nu}^{upper} \simeq 0.7\ eV \tag{90}$$

The decay rate is found to be

$$\Gamma_{\beta\beta} \simeq \frac{Q^5 G_F^4 \bar{m}_{\nu_e}^2 p_F^2}{60\pi^3} \simeq 1.0\times 10^{-61} GeV \tag{91}$$

with the two electron energy $Q \simeq 2$ MeV and $p_F \simeq 50$ MeV.

6. In this case, solar neutrino deficit has to be explained by oscillation between ν_e and a sterile neutrino ν_s [65,14,17] via MSW effects. Since strong bounds on the number of neutrino species both from the invisible Z^0-width and from primordial nucleosynthesis [66,67] require the additional neutrino to be sterile (singlet of SU(2)$\times$ U(1), or singlet of SO(10) in the GUT SO(10) model). Masses and mixings of the triplet sterile neutrinos can be chosen by introducing an additional singlet scalar with VEV $v_s \simeq 336$ GeV. We find

$$\begin{aligned} m_{\nu_s} &= \lambda_H v_s^2/v_{10} \simeq 2.8\times 10^{-3} eV \\ \sin\theta_{es} &\simeq \frac{m_{\nu_L\nu_s}}{m_{\nu_s}} = \frac{v_2}{2v_s}\frac{\epsilon_P}{\epsilon_G^2} \simeq 3.8\times 10^{-2} \end{aligned} \tag{92}$$

with the mixing angle consistent with the requirement necessary for primordial nucleosynthesis [68] given in [66]. The resulting parameters

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq 6.2\times 10^{-6} eV^2, \qquad \sin^2 2\theta_{es} \simeq 5.8\times 10^{-3} \tag{93}$$

are consistent with the values [65] obtained from fitting the experimental data:

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq (4-9)\times 10^{-6} eV^2, \qquad \sin^2 2\theta_{es} \simeq (1.6-14)\times 10^{-3} \tag{94}$$

V. CONCLUSIONS AND REMARKS

Based on the symmetry group SUSY SO(10)$\times\Delta(48)\times$ U(1)[7] , we have presented in much greater detail an alternative interesting model with small $\tan\beta$. The needed nonzero textures are constructed by using the properties of the dihedral group $\Delta(48)$ given by Table 5 via Froggatt and Nielsen mechanism [70] shown in Figs.1 and 2. It is amazing that nature has

[7]Here $\Delta(48)$ is the non-Abelian discrete dihedral group $\Delta(3n^2)$ with $n = 4$, a subgroup of SU(3) [69].

allowed us to make predictions in terms of a single Yukawa coupling constant and three ratios of the VEVs determined by the structure of the physical vacuum and understand the low energy physics from the GUT scale physics. It has also suggested that nature favors maximal spontaneous CP violation. In comparison with the model with large $\tan\beta \sim m_t/m_b$, i.e., Model I, the model analyzed here with low $\tan\beta$, i.e., Model II has provided a consistent picture on the 23 parameters with better accuracy. Besides, ten relations involving fermion masses and CKM matrix elements are obtained with four of them independent of the RG scaling effects. Five relations in the light neutrino sector are also found to be independent of the RG scaling effects. These relations are our main results which contain only low energy observables. As an analogy to the Balmer series formula, these relations may remain to be considered as empirical at the present moment. They have been tested by the existing experimental data to a good approximation and can be tested further directly by more precise experiments in the future. The two types of the models corresponding to the large $\tan\beta$ (Model I) and low $\tan\beta$ (Model II) might be distinguished in testing the MSSM Higgs sector at Colliders as well as by precisely measuring the ratio $|V_{ub}/V_{cb}|$ since this ratio does not receive radiative corrections in both models. The neutrino sector is of special interest for further study. Though the recent LSND experiment, atmospheric neutrino deficit, and hot dark matter could be simultaneously explained in the present model, solar neutrino puzzle can be understood only by introducing an SO(10) singlet sterile neutrino. The scenario for the neutrino sector can be further tested through $(\nu_e - \nu_\tau)$ and $(\nu_\mu - \nu_\tau)$ oscillations since the present scenario has predicted a short wave $(\nu_e - \nu_\tau)$ oscillation. However, the $(\nu_\mu - \nu_\tau)$ oscillation is beyond the reach of CHORUS/NOMAD and E803. As we have also shown that one may abondon the assumption of universality for all terms in the superpotential and use a permutation symmetry among the fields concerning the '22' and '32' textures, consequently, the resulting predictions in the quark and charged lepton sector are unchanged and remain involving four parameters. It is also interesting to note that even if without imposing the permutation symmetry and universality of the coupling constants, the resulting Yukawa coupling matrices of the quarks and leptons only add one additional parameter which can be determined by the charm quark mass. For $m_c(m_c) = 1.27 \pm 0.05$GeV, the resulting predictions remain the same as those in Tables II and III for the quark and charged lepton sector. For the neutrino sector, three additional parameters corresponding to the three nonzero textures are involved. Nevertheless, it remains amazing that nature allows us to make predictions on 23 observables by only using nine parameters for this general case. It is expected that more precise measurements from CP violation, neutrino oscillation and various low energy experiments in the near future could provide an important test on the present model and guide us to a more fundamental theory. It is of interest to note that with gauged SO(3) flavor symmetry, the intriguing bi-maximal mixing scenario can be resulted to understand both solar and atmospheric neutrino data [71].

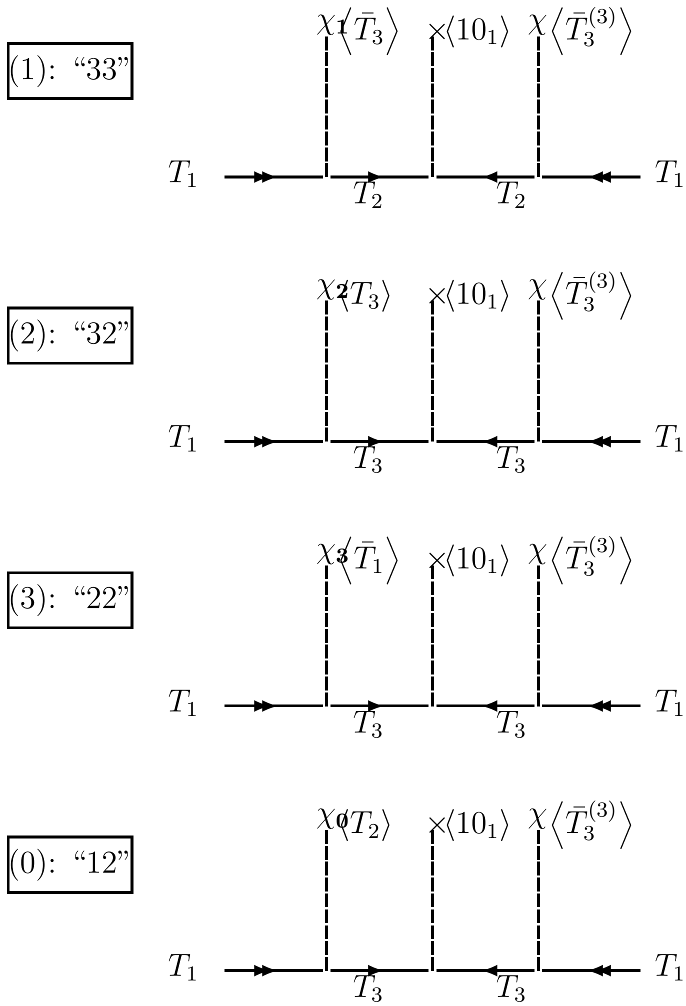

FIG. 1. four non-zero textures resulting from the family symmetry $\Delta(48)$ and U(1) symmetry, are needed for constructing fermion Yukawa coupling matrices.

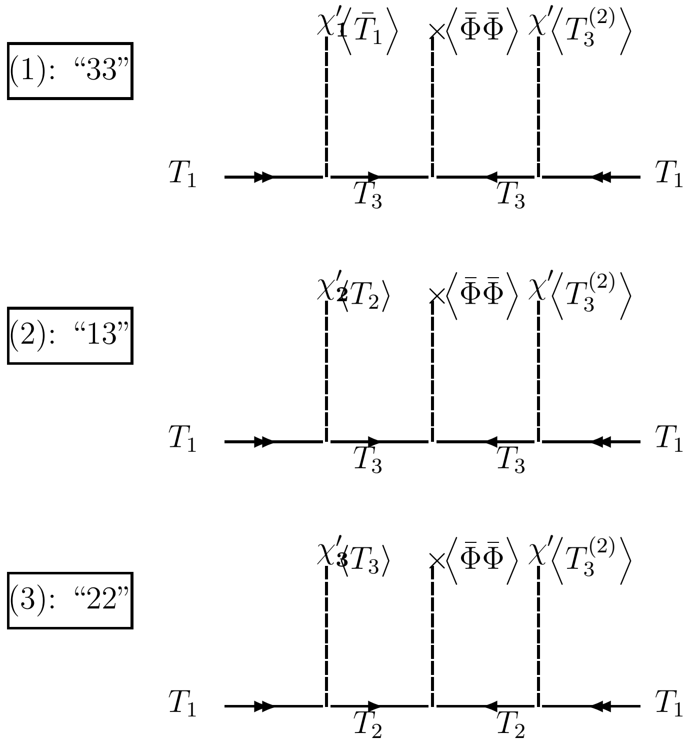

FIG. 2. three non-zero textures resulting from the family symmetry $\Delta(48)$ and U(1) symmetry, are needed for constructing right-handed Majorana neutrino mass matrix.

TABLE V., Decomposition of the product of two triplets, $T_i \otimes T_j$ and $T_i \otimes \bar{T}_j$ in $\Delta(48)$. Triplets T_i and $\bar{T}_i$ are simply denoted by i and $\bar{i}$ respectively. For example $T_1 \otimes \bar{T}_1 = A \oplus T_3 \oplus \bar{T}_3 \equiv A3\bar{3}$, here A represents singlets.

$\Delta(48)$	1	$\bar{1}$	2	3	$\bar{3}$
1	$\bar{1}\bar{1}2$	$A3\bar{3}$	$\bar{1}3\bar{3}$	123	$12\bar{3}$
2	$\bar{1}3\bar{3}$	$13\bar{3}$	$A22$	$1\bar{1}\bar{3}$	$1\bar{1}3$
3	123	$\bar{1}23$	$1\bar{1}\bar{3}$	$2\bar{3}\bar{3}$	$A1\bar{1}$

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